Scaling Laws for Infrastructure Single and Multihop Wireless Networks in Wideband Regimes

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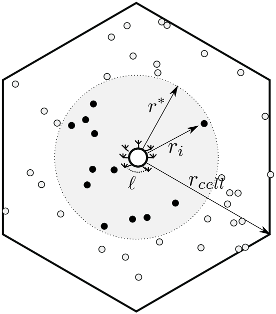
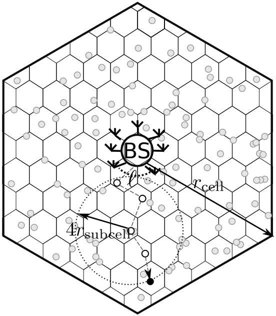
- Relays/multi-hop/cooperation
  - Improve received power at the cost of degrees of freedom.
  - Big disappointment in 4G (gain not worth the DoF loss).
- Future 5G cellular networks
  - Very large bandwidth ($\sim$GHz).
  - High number of antennas.
  - Very high density.
  - Adverse path loss.
- Large bandwidth $\rightarrow$ power limited links $\rightarrow$ underexploited DoF
- **WIN-WIN scenario for 5G multi-hop architecture!**

Contribution: Single and Multi-hop rate scaling laws

Infrastructure Single Hop (ISH)



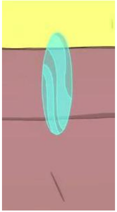
Infrastructure Multi-Hop (IMH)



- Feasible rate vs. number of nodes scaling laws as $n \to \infty$.
- No. BS, antennas, area and **bandwidth** scale with n.
- Three resulting regimes as W increases:
  1. Both limited by W, perform equally.
  2. ISH limited by power and IMH by W, IMH bandwidth gain.
  3. Both limited by power, IMH power gain.

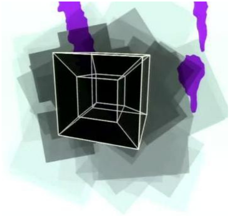
Related work: Point-to-point Wideband Channel

- **"Non-peaky" signal** $\mathrm{E}\left[|X|^4\right] < \infty$.
- No a-priori CSI.
- $C(W)$ capacity as a function of bandwidth.
- $\lim_{W \to \infty} C(W) = 0$ (Medard & Gallager 2002).
- $C(W)$ bell-shaped (Lozano & Porrat 2012).
- Critical bandwith $\frac{P}{N_0}\sqrt{\frac{1}{7}\frac{B_c T_c}{\log(B_c T_c)}} \leq W^* \leq \frac{P}{N_0}\sqrt{7\frac{B_c T_c}{\log(B_c T_c)}}$.



2D

3D



4D

- Capacity region analysis complexity increases with number of dimensions (individual rates of nodes).
  - Define *Feasible rate* $R(n)$ simultaneous to all nodes.
  - Provides a measure of the size of the rate region.
- Study the *scaling exponent* $\log_n(R(n))$ as $n \to \infty$.
  - Find asymptotic *trends* of rate without knowing capacity region completely.
  - Understand fundamental behavior of large networks.

- Dense and extended *ad-hoc* networks. (Gupta & Kumar 2003)
- Forced power-limited regime. (Negi et al 2004)
- Operating Regimes. (Ozgur et al 2009)
- Infrastructure-supported ad-hoc inter-node communications. (Shin et al 2011)

Main differences in our model.

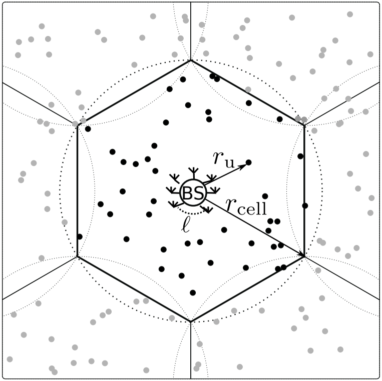
- Infrastructure true cellular BS-node communications.
- $W \to \infty$ and $n \to \infty$, but arbitrary ratio W/n so power-limited regime depends on critical bandwidth.

References vs 5G Properties

Reference	Medard'02	Lozano'12	Gupta'03	Ozgur'09	Shin'12	Negi'04
Wideband	✓	✓	✗	✗	✗	✓
Critical Bandwidth	✗	✓	✗	✗	✗	✗
Scaling laws	✗	✗	✓	✓	✓	✓
Arbitrary area	✗	✗	✗	✓	✗	✗
Infrastructure	✗	✗	✗	✗	✓	✗

- Wireless network with $n \to \infty$ nodes.
- Uniform dist. in area A.
- Served by m Base Stations. Each BS has ℓ antennas.
- **Downlink traffic flows.**
- **Available bandwidth $W \to \infty$, but arbitrary ratio W/n.**

$$\psi := \lim_{n,W\to\infty} \frac{\log W}{\log n}, \qquad (1)$$



Scaling Exponents of Network Parameters

Exponent	Range	Parameter (vs. No. of nodes n)
ψ	$[0, \infty)$	Bandwidth $W = W_0 n^{\psi}$
ν	$[0, 1]$	Area $A = A_0 n^{\nu}$
β	$[0, 1]$	No. of BSs $m = m_0 n^{\beta}$
γ	$[0, 1 - \beta]$	No. of BS antennas $\ell = \ell_0 n^{\gamma}$

Infrastructure Single Hop (ISH) Protocol

- BS direct transmission to node.
- Each cell has $\frac{n}{m} \pm \delta$ nodes w.h.p.
- BS has ℓ transmit antennas.
- BS-node links.
  - MU-MIMO space division mux. $\frac{n}{m\ell}$ users per antenna dim.
  - Orthogonal bandwidth allocation in each dim. $W_{\mathrm{u}} = W\frac{\ell m}{n}$.
  - Uniform power allocation $P_{\mathrm{u}} = P_{\mathrm{BS}}\frac{m}{n}$.
  - Worst case distance $\Theta(n^{(\beta-\nu)})$.

- Subdivide cell in *routing sub-cells* $r_{\mathrm{subcell}} \propto n^{1-\nu}$
- BS uses ℓ antennas and MU-MIMO to initiate as many simultaneous routes.
  - $W_{\mathrm{u}} = W$
  - $P_{\mathrm{u}} = \frac{P_{\mathrm{BS}}}{\ell}$
- One node in each sub-cell forwards data to the next.
  - $W_{\mathrm{u}} = W$
  - $P_{\mathrm{u}} = P$
- Non-scaling (constant) time division scheduling to avoid collisions and satisfy the half-duplex constraint.
- Worst case distance $\Theta(n^{(1-\nu)})$.

- AWGN model for interference and thermal noise.

- Power Spectral Density

$$N_{\mathrm{I}} = \frac{\sum_{i \in \mathcal{I}} P_{\mathrm{I}}}{W \ell_{\mathrm{t}}} + N_0, \tag{2}$$

  - Interferer set $\mathcal{I}$ varies with protocol
  - Interference power spread uniformly in all bandwidth W.
  - Number of antennas per interference transmitter ℓ_{t}.

- As $n \to \infty$ either
  - $\lim_{n \to \infty} \frac{\sum_{i \in \mathcal{I}} P_{\mathrm{I}}}{W \ell_{\mathrm{t}}} = \infty$, N_I dominated by interference.
  - $\lim_{n \to \infty} \frac{\sum_{i \in \mathcal{I}} P_{\mathrm{I}}}{W \ell_{\mathrm{t}}} = 0$, N_I dominated by N_0.

- Transmitter allocates to each user u
  - Tx. bandwidth W_{u}
  - Tx. power P_{u}
  - At distance r_{u} receive power becomes $P_{r_{\mathrm{u}}}$

Lemma

User achieves rate scaling

$$R_{\mathrm{u}} = \begin{cases} \Theta\left(W_{\mathrm{u}}\right) & W_{\mathrm{u}} < W_{\mathrm{u}}^{*} \\ \Theta\left(\frac{P_{r_{\mathrm{u}}}}{N_{\mathrm{I}}}\right) & W_{\mathrm{u}} \geq W_{\mathrm{u}}^{*} \end{cases} \tag{3}$$

- When N_I dominated by interference, $\frac{P_{r_{\mathrm{u}}}}{W_{\mathrm{u}} N_{\mathrm{I}}}$ =const., $R \propto W_{\mathrm{u}}$
- When N_I dominated by noise and $W_{\mathrm{u}} < W_{\mathrm{u}}^{*}$, $\rightarrow R \propto W_{\mathrm{u}}$
- When N_I dominated by noise and $W_{\mathrm{u}} \geq W_{\mathrm{u}}^{*}$, $\rightarrow R \propto \frac{P_{r_{\mathrm{u}}}}{N_{\mathrm{I}}}$

Theorem

Downlink ISH feasible rate per node scales as

$$R_{\mathrm{ISH}}(n) \sim \Theta\left(n^{\beta-1+\min\left(\psi+\gamma,(\beta-\nu)\frac{\alpha}{2}\right)}\right) \tag{4}$$

Proof.

- Noise+Interference PSD N_{I} scales as $\Theta\left(n^{\max\left(\frac{\alpha}{2}(\beta-\nu)-\psi-\gamma,0\right)}\right)$

When $\frac{\alpha}{2}(\beta-\nu)-\psi-\gamma>0$

- Interference dominates.
- Links degrees-of-freedom limited.
- $R(n)=\Theta(W_{\mathrm{u}})$

Otherwise

- Noise dominates.
- $P(W_{\mathrm{u}} \leq W_{\mathrm{u}}^{*}) \to 0$.
- Links power limited.
- $R(n)=\Theta(\frac{P_{\mathrm{u}}}{N_0})$

□

Theorem

Downlink IMH feasible rate per node scales as

$$R_{\mathrm{IMH}}(n) \sim \Theta\left(n^{\beta-1+\min\left(\psi+\gamma,(1-\nu)\frac{\alpha}{2}\right)}\right) \tag{5}$$

Proof.

- Variations of the Noise+Interference PSD argument.
- BS is the bottleneck.

First hop

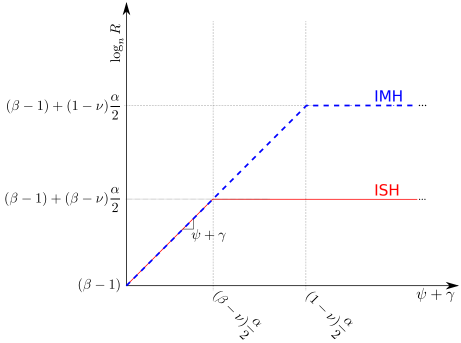
- Broadcast Channel.
- Short subcell range.
- $$\Theta\left(n^{\beta-1+\min\left(\psi+\gamma,(1-\nu)\frac{\alpha}{2}\right)}\right)$$

Second and further

- point-to-point channel.
- Short subcell range.
- $$\Theta\left(n^{\beta-1+\gamma+\min\left(\psi,(1-\nu)\frac{\alpha}{2}\right)}\right)$$

□

Scaling Exponents for downlink ISH and IMH.



- W^* gain $n^{\psi-(\beta-\nu)\frac{\alpha}{2}}$
- Power gain $n^{1-\beta}$

- Regime I:
  Both ISH and IMH exploit DoF.
- Regime II:
  Only IMH exploits DoF.
- Regime III:
  Multi-hop power limited but still superior.

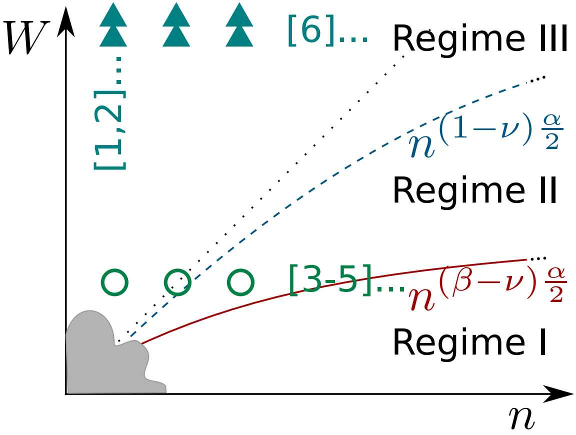


Fig. 1 : Operating regimes and Literature.

- Studied scaling laws of network with bandwith W and size n
- Relation between network size n, critical bandwith W^* and protocol.
  - Single hop becomes power limited when $W > n^{(\beta-\nu)\frac{\alpha}{2}-\gamma}$.
  - Multi hop becomes power limited when $W > n^{(1-\nu)\frac{\alpha}{2}-\gamma}$.
- Linear scaling and protocols are equal for $\beta = 1$.
- For any other values, IMH is superior.
- Networks with very high bandwidth, such as 5G, are likely to need multi-hop to exploit it.

Future work

- Uplink results (easily follows).
- Upper bound of rate scaling.
- Infrastructure relay rodes.
- Hierarchical cooperation with infrastructure.

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Two claims:

1. The exponent of N_{I} for ISH is

$$N_{\mathrm{I,ISH}} \sim \Theta\left(n^{\max\left(\frac{\alpha}{2}(\beta-\nu)-\psi-\gamma,0\right)}\right) \tag{6}$$

2. In ISH asymptotically all links become degrees-of-freedom limited when N_{I} is interference limited, and power limited when it is noise limited. In other words, when N_{I} is dominated by interference (noise) the probability of overspreading tends to zero (one) as $n \to \infty$.

For each node u the interferer set $\mathcal{I}_{\mathrm{ISH}}$ contains all BSs except the one that serves u.

$$N_{\mathrm{I,ISH}} = \frac{1}{W\ell} \sum_{i \in \mathcal{I}_{\mathrm{ISH}}} r_{i,u}^{-\alpha} P_{\mathrm{BS}} + N_0. \tag{7}$$

Lower bound $r_{i,u}$ by the distance between i and the border of the cell. In the hexagonal tessellation of the plane there are $6k$ cells that form a ring at exactly $2k-1$ cell radii from node u.

$$\begin{aligned} \sum_{i \in \mathcal{I}_{\mathrm{ISH}}} r_{i,u}^{-\alpha} &\leq r_{\mathrm{cell}}^{-\alpha} \sum_{k=1}^{\infty} (6k)(2k)^{-\alpha} \\ &\leq 6(2r_{\mathrm{cell}})^{-\alpha} \zeta(\alpha - 1) \end{aligned} \tag{8}$$

where $\zeta(\alpha-1)$ is the Riemann Zeta function evaluated in $\alpha - 1$, which is just some constant for any fixed $\alpha > 2$. This shows that interference power scales as $n^{\frac{\alpha}{2}(\beta-\nu)}$, while noise PSD is constant, so (7) scales as (6).

Critical bandwidth for each user u.

$$W^*(r_\mathrm{u}) \propto \frac{P_\mathrm{BS}}{\ell N_I}(r_\mathrm{u})^{-\alpha} \tag{9}$$

Instead of comparing the bandwidth limitation $W^*(r_\mathrm{u})$ and the actual bandwidth W one user at a time, compute the *critical distance* from the BS, r^*. The fraction of nodes in the cell at less thatn r^* from the BS

$$f_\mathrm{ISH} = \min\left(\frac{2\pi(r^*_\mathrm{ISH})^2}{A_0 n^{\nu-\beta}}, 1\right) \propto n^{\frac{-2(\gamma+\psi)}{\alpha}+(\beta-\nu)}, \tag{10}$$

converges to one when $\frac{-2(\gamma+\psi)}{\alpha} + (\beta-\nu) > 0$ (interference-limited case), and to zero otherwise (noise-limited case).

Sketch of proof of Theorem 2

1st hop is a BC channel, 2nd and following are point to point channels. Same analysis as in ISH with some modified parameters holds per hop.

- $\psi + \gamma < \frac{\alpha}{2}(1-\nu)$, all links are degrees-of-freedom-limited.
- $\psi < \frac{\alpha}{2}(1-\nu) < \psi + \gamma$, first-hop links are power-limited, the following are interference limited. The fraction of nodes that are not overspread in the first hop converges to zero and overspreading affects the allocation of ℓW antennas and bandwidth resources at the BS, but not usage of bandwidth W when nodes transmit.
- $\psi > \frac{\alpha}{2}(1-\nu)$, all links are power-limited. The fraction of nodes that are not overspread nodes in second and following hops converges to zero and overspreading affects all uses of bandwidth.

Comparing the rates in all regimes shows that the bottleneck is always the first hop. Combining, we obtain Theorem 3.

Theorem 2, first hop

BS uses ℓ antennas and MU-MIMO to initiate as many simultaneous routes.

- $W_{\mathrm{u}} = W$
- $P_{\mathrm{u}} = \frac{P_{\mathrm{BS}}}{\ell}$
- $N_{\mathrm{I,IMH}} \sim \Theta\left(n^{\max\left(\frac{\alpha}{2}(1-\nu)-\psi-\gamma,0\right)}\right)$

Using (3), feasible rates in the first hop are $\Theta(W_{\mathrm{u}}) = \frac{m\ell}{n} W$ in the first regime and $\Theta(\frac{P_{r_{\mathrm{u}}}}{N_{\mathrm{I}}}) \propto n^{\beta-1+\frac{\alpha}{2}(1-\nu)}$ in the other two.

One node in each sub-cell forwards data to the next.

- $W_{\mathrm{u}} = W$
- $P_{\mathrm{u}} = P$
- $N_{\mathrm{I,IMH}} \sim \Theta\left(n^{\max\left(\frac{\alpha}{2}(1-\nu)-\psi,0\right)}\right)$

Feasible rates are $\Theta(W_{\mathrm{u}}) = W$ in the two first regimes, and $\Theta(\frac{P_{r_{\mathrm{u}}}}{N_{\mathrm{I}}}) \propto n^{\frac{\alpha}{2}(1-\nu)}$ in the last regime. However, these rates are obtained per route and not per node, so they must normalized multiplying by ℓ simultaneous routes and dividing by $\frac{n}{m}$ nodes per cell.

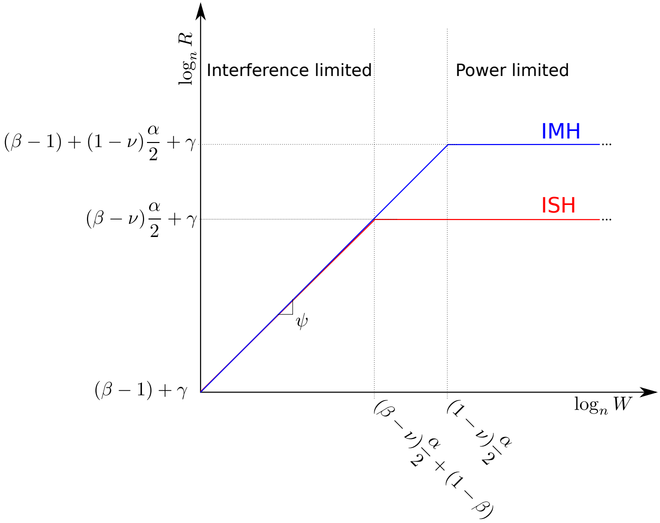

Fig. 2 : Scaling exponents for uplink ISH and IMH.

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